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D-branes and orientifolds of SO(3)

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Abstract: We study branes and orientifolds on the group manifold of SO(3). We consider particularly the case of the equatorial branes, which are projective planes. We show that a Dirac-Born-Infeld action can be defined on them, although they are not orientable. We find that there are two orientifold projections with the same spacetime action, which differ by their action on equatorial branes.

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Contents

1. Introduction

The purpose of this note is to study branes and orientifolds on the group manifold of SO(3) in a geometrical way. The group SO(3) is the quotient of SU(2) by its center $\mathbb{Z}_2$, whose non-trivial element acts as $g \to -g$, which is geometrically the antipodal map; thus the geometry of SO(3) is $\mathbb{RP}^3$.

Strings on SO(3) are described by a SU(2) WZW model with k even. Their torus amplitude, derived in [1], is given by the $D_{k/2+2}$ modular invariant. The spectrum of oriented closed strings thus contains non-twisted states, whose left and right isospins j and $\bar{j}$ are equal and integer, and twisted states with $\bar{j} = k/2 - j$, where j is integer if k is a multiple of 4, and half-integer otherwise. In particular, there always exists a twisted state with $j = \bar{j} = k/4$ and this is the only one with $j = \bar{j}$.

First, we consider D-branes on SO(3). They were studied in various papers, in a purely algebraic way [3, 4] or in a more geometric manner [6, 7, 8].

Like in any WZW model, the maximally symmetric branes (i.e. those that preserve the Kac-Moody symmetry) are localized on conjugacy classes of the group. What is particularly interesting here is the conjugacy class located at the "equator" (the rotations of angle π), which is a projective plane. This gives rise to two sorts of $\mathbb{RP}^2$ branes, which differ only by their coupling to twisted closed strings; they can be combined in pairs into wrapped spherical branes.

We also show that a Dirac-Born-Infeld action can be defined on the $\mathbb{RP}^2$ brane, although it is not orientable. Then we compute the spectrum of its small fluctuations like in [9], and we find that it corresponds to the spectrum of the open strings, as expected.

Next, we consider the orientifolds. They were studied purely algebraically in [3, 4], and only recently in a geometric way: [12, 13] study them on general WZW models, and [14] studies in great detail the case of the three-sphere, including the relations between branes and orientifolds. Here we will study the SO(3) case in a similar manner.

We find that there are two orientifold projections with the same orientifold fixed points. These projections essentially differ by their actions on equatorial branes: in one case, the projection acts separately on the two sorts of $\mathbb{RP}^2$ branes; in the other case, only spherical branes exist and the projection acts on them as a whole.

All the results found here geometrically are consistent with the previous algebraic results.

2. The D-branes and the annulus amplitude

2.1 Geometrical description of the D-branes

The simplest way of studying D-branes on SO(3) is by describing them as branes of SU(2) with the $\mathbb{Z}_2$ antipodal identification.

On SU(2), which is simply connected, maximally symmetric branes are Cardy states [2]; geometrically, the Cardy state associated with the representation of isospin j is a sphere, whose equation in the usual spherical coordinates is $\psi = (2j+1)\pi/(k+2)$ [5, 6]. Now the antipodal identification acts as

$$\psi \to \pi - \psi\,, \qquad \theta \to \pi - \theta\,, \qquad \phi \to \phi + \pi\,, \tag{2.1}$$

which transforms the Cardy state of isospin j into the Cardy state of $k/2 - j$.

So, for $j < k/4$, the two distinct branes corresponding to the Cardy states of isospin j and $k/2 - j$ of SU(2) are identified; thus, one obtains spherical branes, which will be called "branes of type j".

On the other hand, on the "equator", which is a projective plane because of the $\mathbb{Z}_2$ identification, one can have two sorts of branes:

- by identifying a point on an equatorial brane of S^3 with the opposite point on *another* equatorial brane, one obtains a spherical brane wrapped on the equator (i.e. on any distinct point of the equator there are two points of the brane).
- by identifying a point on an equatorial brane of S^3 with the opposite point on the *same* brane, one obtains a projective plane brane.

More precisely, to derive the spectrum and the gauge group concerning the equatorial branes, we will use the following description: we have N equatorial spheres on S^3, and the $\mathbb{Z}_2$ identification acts as an involutive permutation of them. Now a boudary state can be an arbitrary linear combination of branes, so the space of boundary states is a N-dimensional vector space and $\mathbb{Z}_2$ acts on it as a $N \times N$ unitary matrix Z. $Z^2 = 1$, so it is diagonalizable with n_+ eigenvalues $+1$ and n_- eigenvalues -1, with $n_+ + n_- = N$, i.e. our branes can be described as n_+ $\mathbb{RP}^2$ branes with sign $+$, and n_- $\mathbb{RP}^2$ branes with sign $-$. Thus the equatorial brane of the three-sphere gives rise to two sorts of branes (this was observed in [6]). An interpretation of these signs will be given in the next section.

Now spherical equatorial branes correspond to $Z = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, which has one $+1$ and one -1 eigenvalue, so a spherical equatorial brane is a combination of two $\mathbb{RP}^2$ branes of opposite signs.

2.2 The annulus amplitude

Strings between a D-brane of type r and one of type s correspond in the 3-sphere to strings between branes of isospins r and s on one hand, and $k/2 - r$ and s on the other hand (the two other possibilities are identified with these ones). So the corresponding annulus reads:

$$\mathcal{A}_{rs} = \sum_{l=0}^{k/2} (N_{rs}{}^{l} + N_{k/2-r,s}{}^{l})\, \chi_l(\sqrt{q}) \,, \tag{2.2}$$

where $q = e^{-2\pi t}$, χ_l are the Kac-Moody characters, and $N_{rs}{}^{l}$ are the fusion coefficients of the SU(2) Kac-Moody algebra, whose non-zero values are

$$N_{rs}{}^{l} = 1, \;\text{ for } l = |r-s|, |r-s|+1, \ldots, \min(r+s, k-r-s) \,. \tag{2.3}$$

(l can be integer or half-integer except when stated otherwise.)

In order to extract the D-brane couplings to the closed-string modes we must rewrite these amplitudes in the dual 'transverse' channel. This is achieved by changing variables

$$q = e^{-2\pi t} \to \tilde{q} = e^{-2\pi/t} \,, \tag{2.4}$$

and using the modular property of the characters

$$\chi_l(\sqrt{q}) = S_l{}^{j}\, \chi_j(\tilde{q}^2) \,. \tag{2.5}$$

Consistency requires the final result to be of the general form

$$\mathcal{A}_{rs} = \sum_{j=0}^{k/2} D_r^j D_s^j \,\chi_j(\tilde{q}^2) \,, \tag{2.6}$$

where D_s^j is the coupling of a D-brane of type s to closed strings in the (j, j) representation of the Kac-Moody algebra.

The modular-transformation matrix for the WZW model reads:

$$S_l{}^{j} = \sqrt{\frac{2}{k+2}} \sin\left(\frac{(2l+1)(2j+1)\pi}{k+2}\right) . \tag{2.7}$$

One then finds

$$D_s^j = E_{2j}\sqrt{2}\, \sin\left(\frac{(2j+1)(2s+1)\pi}{k+2}\right) \sqrt{\mathcal{N}_j} \,, \tag{2.8}$$

where E_n is a projector onto even integers (1 if n is even, 0 otherwise), and $\mathcal{N}_j$ is a normalization such that

$$(\mathcal{N}_j)^{-1} \equiv \sqrt{\frac{k+2}{2}} \sin\left(\frac{(2j+1)\pi}{k+2}\right) . \tag{2.9}$$

Thus, up to a factor $\sqrt{2}$ (and a E_{2j} because (j, j) untwisted strings with j half-integer do not exist), these couplings are the same as in the SU(2) case [14].

Note that, for topological reasons, the spherical branes cannot couple to twisted closed strings, so that these are couplings to untwisted closed strings only. Moreover, as spherical equatorial branes are quite similar to other spherical branes, they can be considered in this calculation as branes of type $k/4$, so their couplings are equal to $D^j_{k/4}$.

For an open string stretching between an $\mathbb{RP}^2$ D-brane and one of type s the annulus reads:

$$\mathcal{A}_{Rs} = \sum_{l=0}^{k/2} N_{k/4,s}{}^{l}\,\chi_l(\sqrt{q}) \tag{2.10}$$

(with R standing for $\mathbb{RP}^2$). Note that this is independent of the sign of the $\mathbb{RP}^2$ brane. One then finds

$$D^j_R = \frac{1}{2} D^j_{k/4} \tag{2.11}$$

which is not surprising, since spherical equatorial branes are a combination of two $\mathbb{RP}^2$ branes. Note that this amplitude does not contain any information about couplings of the $\mathbb{RP}^2$ brane with twisted strings.

Now we consider strings stretching between two $\mathbb{RP}^2$ branes of definite sign, which correspond on the 3-sphere to strings between equatorial branes. As such states are eigenvalues of the action of $\mathbb{Z}_2$, which we will also call Z, one half of the states are projected out. One has

$$Z|N,l,ij\rangle = (-)^l s_i s_j |N,l,ij\rangle \text{ for } l = 0,1,\ldots,k/2\,, \tag{2.12}$$

where N is the excitation number of the state, l the spin of the SO(3) representation under which it transforms, i and j the indices of the two branes, s_i and s_j their signs. The range of l corresponds to the non-zero values of $N_{k/4,k/4}{}^{l}$ and the sign $(-)^l$ is the parity of the lth spherical harmonic. So we have only strings of even isospin between branes of the same sign, and of odd isospin between branes of opposite sign. Thus, the corresponding annulus amplitude reads

$$\mathcal{A}_{RR} = \frac{1}{2} \sum_{\substack{l=0 \\ l \text{ integer}}}^{k/2} \chi_l(\sqrt{q}) \pm \frac{1}{2} \sum_{\substack{l=0 \\ l \text{ integer}}}^{k/2} (-)^l \chi_l(\sqrt{q})\,, \tag{2.13}$$

where $\pm$ is the product of both signs.

As the first term in the latter sum contains all the couplings of the $\mathbb{RP}^2$ brane with untwisted strings, the second term contains only the couplings with twisted strings. After a modular transformation, one finds that the brane couples only with the twisted state with $(j,\bar{j}) = (k/4,k/4)$, and the coupling is

$$D_R^{\text{twisted}} = \pm\frac{1}{2}\sqrt{\frac{k+2}{2}}\,, \tag{2.14}$$

where $\pm$ is the sign of the brane. So we see that the sign of a brane can be interpreted, up to a numerical factor, as its charge under the twisted state with $(j,\bar{j}) = (k/4,k/4)$ (this was already noticed in [7]). Note that a spherical equatorial brane has thus a vanishing charge under this twisted state, which is normal since its topology does not allow any coupling between them.

Now let us compute the gauge symmetry group. As photons are strings with both endpoints on branes of the same type, and the same twisted charge for $\mathbb{RP}^2$ branes, the gauge group is $\prod_{j<k/4} \mathrm{U}(n_j) \times \mathrm{U}(n_+) \times \mathrm{U}(n_-)$. In particular, for s spherical equatorial branes and r $\mathbb{RP}^2$ branes of the same charge (which is the general case since $\mathbb{RP}^2$ branes of opposite charges can be combined into spherical branes), the gauge group is $\mathrm{U}(s+r)\times\mathrm{U}(s)$.

Note that it is not clear whether or not a configuration with several branes is stable; for example, in the SU(2) case, it was shown (see for example [15]) that such a configuration can decay into a single brane. We will not discuss stability considerations in this paper.

2.3 Semi-classical considerations

Some of the results found here can be derived from a semi-classical analysis, similar to what was done in [9] for the 3-sphere.

In the case of spherical branes, we can apply the general results of [10], and we find that the energy of the brane and the fluctuations are essentially the same than on the 3-sphere. For the spherical equatorial brane, the spectrum of small fluctuations contains only half of the light open strings because this brane is wrapped on a projective plane; an interpretation in terms of a non-commutative geometry is given in [8].

In the case of equatorial $\mathbb{RP}^2$ branes, one may think that the non-orientability of the brane makes the definition of a Born-Infeld action impossible. In fact, this is not the case: to define the integration of a real function on $\mathbb{RP}^2$, one just needs a measure; since the measure $d\theta\, d\phi$ on S^2 is $\mathbb{Z}_2$-invariant, one obtains a well-defined measure for $\mathbb{RP}^2$. The question is then to check whether the integrand, i.e. the Born-Infeld lagrangian, is invariant. Without fluctuations, and taking fluxes $F = 0$ and $B = 0$ (a two-form cannot have a non-vanishing constant flux on a non-orientable surface), the lagrangian just contains the metric, so the invariance is trivial, making the Born-Infeld action well-defined. Thus the energy of the $\mathbb{RP}^2$ brane is simply one half of the energy for an equatorial spherical brane, which is not surprising.

Now consider small fluctuations of this brane. We write them like in [9], parametrizing the worldvolume by (t, θ, ϕ):

$$\psi = \frac{\pi}{2} + \delta\,, \qquad A_\theta = \frac{k}{2\pi}\alpha_\theta \qquad \text{and} \qquad A_\phi = \frac{k}{2\pi}\alpha_\phi\,. \tag{2.15}$$

Then the computation of the linearized equation of motion is straightforward, and one finds

$$\frac{d^2}{dt^2}\begin{pmatrix}\delta \\ f/\sin\theta\end{pmatrix} = -\frac{1}{k\alpha'}\begin{pmatrix}\Box + 2 & 2 \\ 2\Box & \Box\end{pmatrix}\begin{pmatrix}\delta \\ f/\sin\theta\end{pmatrix} \tag{2.16}$$

with $f = \partial_\theta \alpha_\phi - \partial_\phi \alpha_\theta$.

Now the solutions we are looking for must be invariant under the transformations (2.1), i.e. one must have

$$\begin{aligned}\delta(\pi-\theta, \phi+\pi) &= -\delta(\theta,\phi) \\ A_\theta(\pi-\theta, \phi+\pi) &= -A_\theta(\theta,\phi) \\ A_\phi(\pi-\theta, \phi+\pi) &= A_\phi(\theta,\phi)\,.\end{aligned} \tag{2.17}$$

This implies that δ and $f/\sin\theta$ are odd functions, so they contain only odd spherical harmonics. Finally, after some more computations, the spectrum of quadratic fluctuations is

$$m^2 = \frac{l(l+1)}{k\alpha'} \quad \text{in reps. } (l-1) \oplus (l+1) \text{ with } l \text{ even.} \tag{2.18}$$

Thus the DBI spectrum is the same than the exact CFT result, except for the fact that the former does not have any level-dependent truncation.

3. Orientifolds on SO(3) and the Klein bottle amplitude

As explained in [14], the orientifold action is an orientation reversal Ω on the world-sheet (i.e. $\Omega : z \to \bar{z}$), which must be combined with an orientation-flipping isometry h of the target space to preserve the Wess-Zumino-Witten term in the action. Such an isometry can be described as an isometry of S^3 with $h^2 = \pm 1$. As the isometries with $h^2 = -1$ preserve the orientation, the allowed transformations are the same than on the three-sphere, i.e. $g \to \pm g^{-1}$. As these two possible isometries are identified through $\mathbb{Z}_2$, we are left with only one, whose fixed points are an $\mathcal{O}0$ at the pole (the two poles of S^3 are identified) and a projective plane $\mathcal{O}2$ at the "equator".

Now we can derive the Klein bottle amplitude. To do so, we have to know how the various states transform, more precisely those with $j = \bar{j}$ since only those contribute. The untwisted states are the same than on the three-sphere with the constraint that j must be an integer, so they are invariant under Ωh. The twisted states with $j = \bar{j} = k/4$ also contribute, and since we do not know a priori how they transform, we consider both possibilities. Thus, the Klein bottle amplitude reads

$$\mathcal{K} = \sum_{\substack{l=0 \\ l \text{ integer}}}^{k/2} \chi_l(q^2) + \zeta \chi_{k/4}(q^2) \qquad \text{with } \zeta = \pm 1\,. \tag{3.1}$$

After a modular transformation with the S matrix (eq. (2.7)), the amplitude reads

$$\mathcal{K} = \sum_{j=0}^{k/2} (C^j)^2 \chi_j(\sqrt{\tilde{q}}) \tag{3.2}$$

where C^j is the coupling of the orientifold with the untwisted[1] closed strings. These 'crosscap coefficients' are

$$C^j = \varepsilon_j E_{2j} \sqrt{\mathcal{N}_j} \left(\sin\left(\frac{(2j+1)\pi}{2k+4}\right) + \zeta\,(-)^j \cos\left(\frac{(2j+1)\pi}{2k+4}\right)\right). \tag{3.3}$$

Notice that this coefficient is similar to the sum of the coefficients for the two sorts of orientifold projections of the three-sphere, which meshes nicely with the fact that we have an $\mathcal{O}0$ and an $\mathcal{O}2$ together.

[1] A priori, one may expect that there be also a nonvanishing coupling with twisted strings. This is inconsistent with the Möbius amplitudes

4. Unoriented open strings and the Möbius amplitude

4.1 Unoriented open strings

Unoriented open strings are open strings which are invariant under Ωh, which transforms a brane into a brane of the same type (but not necessarily of the same twisted charge in the case of $\mathbb{RP}^2$ branes) and exchanges the ends of the string. If a string has its ends on branes of different types, a linear combination of that string and its image by Ωh will remain. So we are interested in how Ωh acts on the open strings between branes of the same type.

Strings between non-equatorial branes of type r on SO(3) correspond to two inequivalent sorts of strings on SU(2): strings with both ends on a brane of type r, and strings with one end on a brane of type r and the other on a brane of type $k/2 - r$. The action of Ωh on these strings has been derived in [14]; its eigenvalue for a string of isospin l is proportional to $(-)^l$ in the former case, and independant of l in the latter case. The gauge group for n_r branes is then $\mathrm{SO}(n_r)$ or $\mathrm{USp}(n_r)$, as usual.

In the case of equatorial $\mathbb{RP}^2$ branes, things are not so simple, since branes of opposite charges may be mixed by the orientifold. The most general action of the orientifold on the states is [11]

$$\Omega h|N, l, ij\rangle = (-)^{N+1}\gamma_{ii'}\gamma^*_{jj'}|N, l, j'i'\rangle \tag{4.1}$$

where γ is a unitary, symmetric or antisymmetric, matrix. Now, on SO(3), states must be invariant under the Z transformation (2.12), so we impose that Ωh acting on a Z-invariant state results in a Z-invariant state. This implies that γ must either commute or anticommute with Z.

If γ commutes with Z, $+$ and $-$ branes are not mixed by the orientifold, and the gauge group is $\mathrm{SO}(n_+) \times \mathrm{SO}(n_-)$ or $\mathrm{USp}(n_+) \times \mathrm{USp}(n_-)$.

If γ anticommutes with Z, which is possible only if $n_+ = n_- \equiv n$, then, through changes of basis in the $+$ and $-$ branes separately (i.e. that do not mix $+$ and $-$ branes), γ can be put into the form

$$\gamma = \begin{pmatrix} 0 & \pm\mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}, \tag{4.2}$$

where the first rows and columns of the matrix correspond to $+$ branes. So the effect of Ωh on the Chan-Paton coefficients is

$$\begin{aligned} |++\rangle &\leftrightarrow |--\rangle \\ |+-\rangle &\to \pm|+-\rangle \\ |-+\rangle &\to \pm|-+\rangle\,. \end{aligned} \tag{4.3}$$

Before orientifolding, one had one $\mathrm{U}(n)$ group corresponding to $|++\rangle$ states, and another for the $|--\rangle$ states. As they are identified through the orientifold, one is left with a $\mathrm{U}(n)$ gauge group. Notice that $+$ and $-$ $\mathbb{RP}^2$ branes cannot exist separately here, and that the γ matrix defines pairs of branes of opposite charges, so that it is better to think of them as n spherical equatorial branes. The $\mathrm{U}(n)$ symmetry group is then interpreted as arbitrary changes of basis in the space of these branes.

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4.2 The Möbius amplitude

As explained clearly in [4], the Möbius amplitude is given by a linear combination of $\chi_l(-\sqrt{q})$ in the direct channel, and $\chi_j(-\sqrt{\tilde{q}})$ in the transverse channel. Thus, it is convenient to work in the basis of real characters

$$\hat{\chi}_j(q) \equiv e^{-i\pi(h_j - c/24)}\, \chi_j(-\sqrt{q}) \tag{4.4}$$

The modular transformation to be applied here is then given by the matrix

$$P = T^{1/2} S T^2 S T^{1/2}\,. \tag{4.5}$$

The calculation of this matrix was performed in [4] and can be found in the appendix of [14]. It is shown there that

$$P_l^{\,j} = \frac{2}{\sqrt{k+2}} \sin\left(\frac{\pi(2l+1)(2j+1)}{2(k+2)}\right) E_{2l+2j+k}\,. \tag{4.6}$$

The Möbius amplitude for n_r spherical branes of type r reads

$$\mathcal{M}^r = n_r \varepsilon_r' \sum_{\substack{l=0 \\ l \text{ integer}}}^{2r} \left((-)^l \hat{\chi}_l(q) + \varepsilon_r''\, \hat{\chi}_{k/2-l}(q)\right) \tag{4.7}$$

where ε_r' is a global sign, as always in Möbius amplitudes, the two terms correspond to the two sorts of strings mentioned in the previous subsection, and ε_r'' is for the projection of strings of the second type.

Then, after a modular transformation with the P matrix, the consistency condition

$$\mathcal{M}^r = n_r \sum_j C^j D_r^j \hat{\chi}_j(\tilde{q}) \tag{4.8}$$

implies that ε_j is a constant ε, $\varepsilon_r' = (-)^{2r}\varepsilon$, and $\varepsilon_r'' = (-)^{2r}\zeta$.

As the photons are in the isospin zero representation of the Kac-Moody algebra, the gauge group is given by the sign before χ_0, i.e. ε_r'. So, as for the $\mathcal{O}0$ of the 3-sphere, one has an alternance of orthogonal and symplectic groups.

For the equatorial branes, one has to distinguish between whether γ commutes or anticommutes with Z. In the first case, the states that appear in the Möbius amplitude are $|++\rangle$ and $|--\rangle$, which are of even isospin, so the Möbius amplitude reads

$$\begin{aligned}
\mathcal{M}^R &= (n_+ + n_-)\, \varepsilon_R' \sum_{\substack{l=0 \\ l \text{ even}}}^{k/2} \hat{\chi}_l(q) \\
&= \frac{1}{2}(n_+ + n_-)\, \varepsilon_R' \sum_{\substack{l=0 \\ l \text{ integer}}}^{k/2} \Big((-)^l \hat{\chi}_l(q) + \hat{\chi}_{k/2-l}(q)\Big)\,.
\end{aligned} \tag{4.9}$$

In the second case, the states that appear in the Möbius amplitude are $|+-\rangle$ and $|-+\rangle$, which are of odd isospin, so the Möbius amplitude reads

$$\begin{aligned}\mathcal{M}^R &= 2n\,\varepsilon'_R \sum_{\substack{l=0\\ l\ \mathrm{odd}}}^{k/2} \hat{\chi}_l(q) \\ &= -n\,\varepsilon'_R \sum_{\substack{l=0\\ l\ \mathrm{integer}}}^{k/2} \Big((-)^l \hat{\chi}_l(q) - \hat{\chi}_{k/2-l}(q)\Big)\,. \end{aligned} \tag{4.10}$$

In both cases, up to a factor 1/2, which is also present in the couplings D^j_R (2.11), this amplitude has the same form as the other amplitudes with ε''_R equal to $+1$ in the first case, and -1 in the second case. The consistency condition is then the same as for spherical branes, i.e. $\varepsilon'_R = \varepsilon''_R\,(-)^{k/2}\varepsilon$ and $\zeta = \varepsilon''_R\,(-)^{k/2}$. Thus:

- If $\zeta = (-)^{k/2}$, then the gauge group is $\mathrm{SO}(n_+) \times \mathrm{SO}(n_-)$ or $\mathrm{USp}(n_+) \times \mathrm{USp}(n_-)$, depending on ε_R.
- If $\zeta = -(-)^{k/2}$, then one must have $n_+ = n_- \equiv n$, and the gauge group is $\mathrm{U}(n)$.

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